Unit 5: Day 1: Three Models: Graphic, Numeric, and Algebraic

75 min.

Description

- Develop an understanding of collecting like terms and expanding polynomials through guided exploration.
- Use multiple representations: graphic, numeric and algebraic models.

Materials

- graphing calculators
- BLM 5.1.1, 5.1.2, 5.1.3, 5.1.4, 5.1.5, 5.1.6, 5.1.7
- grid paper

Assessment Opportunities

Minds On ...

Jigsaw (Home Groups of 4) → Discussion

Introduce the final unit of the course outlining the concepts to be studied (algebra and polynomials) and the structure (review the concepts and skills from the earlier units) within which they will be learned.

Emphasize the importance of consolidating learning, making connections and recognizing general patterns. This lesson overlays multiple representations for equations with new algebra skills.

Organize students into home groups of four and designate them as A, B, C, or D within the group.

Action!

Jigsaw (Expert Groups) → Guided Exploration

Divide class into four expert groups: A and B – expansion of an expression, and C and D - collecting like terms. Distribute grid paper and BLM 5.1.1 to Group A, BLM 5.1.2 to Group B, BLM 5.1.3 to Group C, and BLM 5.1.4 to Group D.

The questions require facility with integers. Provide calculators for students who have numeric difficulties.

Students work through the guided discovery and compare their individual conclusions with others in their work group.

Jigsaw (Home Group) → Sharing

They meet with their home group and check conclusions with their "partner" with the group – A with B and C with D in preparation for sharing their learning with the other pair.

You may wish to predetermine strong/weak pairs.

Curriculum Expectations/Observation/Mental Note: Pairs share with each other what they have learned and check for understanding. All members of the home group record the learning from both tasks.

Consolidate Debrief

Whole Class → Guided Connections

Guide the class through BLM 5.1.5 to consolidate new expansion and collections skills and to connect algebraic expressions to measurement formulas. Clarify any misunderstandings observed while pairs were working together.

Home Activity or Further Classroom Consolidation

Application
Concept Practice

Complete the worksheets 1.6 and 1.7.

Complete textbook problems requiring the collection of like terms and monomial times a binomial. (The teacher assigns appropriate exercises.)

5.1.1: Three Models: Graphic, Numeric, and Algebraic

Using a graphing calculator or spreadsheet, create a table of values for:

a) $y = 3(x - 1)$

x	*y*
−2	
−1	
0	
1	
2	

b) $y = 3x - 3$

x	*y*
−2	
−1	
0	
1	
2	

How do the tables compare?

Using a graphing calculator or graphing software:
- Graph $y = 3(x - 1)$. Record the graph, on the grid paper.
- Graph $y = 3x - 3$ and record the graph, in another colour, on the same grid paper.
- How do the graphs compare?

The tables of values in a) and the graphs in b) should be identical.
- What must this mean about the expressions $3(x - 1)$ and $3x - 3$?

- What process would transform $3(x - 1)$ into $3x - 3$?

On the back of this paper, create tables of values and compare them for:

$y = 2(x + 4)$ $y = 2x + 8$
- What process would transform $2(x + 4)$ into $2x + 8$?

Graph $y = -4(x + 3)$ and $y = -4x - 12$ on the same axes and compare the graphs.
- What process would transform $-4(x + 3)$ into $-4x - 12$?

The process is called “expansion” or “removal of brackets.”
Expand the following:

a) $2(x - 5)$

b) $5(x + 1)$

c) $4(3x - 1)$

d) $-3(2x + 4)$

e) $2(4x - 5)$

f) $-5(x + 4)$

5.1.2: Three Models: Graphic, Numeric, and Algebraic

Using a graphing calculator or spreadsheet, create a table of values for:

a) $y = 4(x - 2)$

x	*y*
−2	
−1	
0	
1	
2	

b) $y = 4x - 8$

x	*y*
−2	
−1	
0	
1	
2	

How do the tables compare?

Using a graphing calculator or graphing software:
- Graph $y = 4(x - 2)$. Record the graph on the grid paper.
- Graph $y = 4x - 8$ and record the graph, in another colour, on the same paper.
- How do the graphs compare?

The tables of values in a) and the graphs in b) should be identical.
- What must this mean about the expressions $4(x - 2)$ and $4x - 8$?

- What process would transform $4(x - 2)$ into $4x - 8$?

On the back of this paper, create tables of values and compare them for:
$y = 3(x + 1)$ $y = 3x + 3$
- What process would transform $3(x + 1)$ into $3x + 3$?

Graph $y = -2(x + 5)$ and $y = -2x - 10$ on the same axes and compare the graphs.
- What process would transform $-2(x + 5)$ into $-2x - 10$?

The process is called “expansion” or “removal of brackets.”
Expand the following:

a) $3(x - 2)$

b) $-5(x + 4)$

c) $6(2x - 1)$

d) $-3(x - 5)$

e) $2(3x + 5)$

f) $-5(2x - 4)$

5.1.3: Three Models: Graphic, Numeric, and Algebraic

Using a graphing calculator or spreadsheet, create a table of values for:

a) $y = 1 + 2x + 3 + 4x$ b) $y = 6x + 4$

x	y
−2	
−1	
0	
1	
2	

x	y
−2	
−1	
0	
1	
2	

How do the tables compare?

Using a graphing calculator or graphing software:
- Graph $y = 1 + 2x + 3 + 4x$. Record the graph on the grid paper.
- Graph $y = 6x + 4$ and record the graph, in another colour, on the same paper.
- How do the graphs compare?

The tables of values in a) and the graphs in b) should be identical.
- What must this mean about the expressions $1 + 2x + 3 + 4x$ and $6x + 4$?

- What process would transform $1 + 2x + 3 + 4x$ into $6x + 4$?

On the back of this paper, create tables of values and compare them for:
$y = 5x^2 + 4 - 3x^2 - 5$ $y = 2x^2 - 1$
- What process would transform $5x^2 + 4 - 3x^2 - 5$ into $2x^2 - 1$?

Graph $y = 3 + 6x - 5 - 3x$ and $y = 3x - 2$ on the same axes and compare the graphs.
- What process would transform $3 + 6x - 5 - 3x$ into $3x - 2$?

The process is called “collection of like terms” or “simplifying.”
Simplify the following:

a) $x + 4x - 8x - 9$

b $-2 + 5 - 4x - 2x$

c) $9 - 1x - 5x - 3$

d) $2 - 2x + 4 - 2x$

e) $3x^2 - 4x + 2 + x^2 + 6x - 2$

5.1.4: Three Models: Graphic, Numeric, and Algebraic

Using a graphing calculator or spreadsheet, create a table of values for:

a) $y = 1x + 2 + 3x + 4$ b) $y = 4x + 6$

x	*y*
−2	
−1	
0	
1	
2	

x	*y*
−2	
−1	
0	
1	
2	

How do the tables compare?

Using a graphing calculator or graphing software

- Graph $y = 1x + 2 + 3x + 4$. Record the graph on the grid paper.
- Graph $y = 4x + 6$ and record the graph, in another colour, on the same paper.
- How do the graphs compare?

The tables of values in a) and the graphs in b) should be identical.

- What must this mean about the expressions $1x + 2 + 3x + 4$ and $4x + 6$?

- What process would transform $1x + 2 + 3x + 4$ into $4x + 6$?

On the back of this paper, create tables of values and compare them for:

$y = 3x^2 - 4 + 5x^2 - 5$ $y = 8x^2 - 9$

- What process would transform $3x^2 - 4 + 5x^2 - 5$ into $8x^2 - 9$?

Graph $y = 2x + 5 + x - 7$ and $y = 3x - 2$ on the same axes and compare the graphs.

- What process would transform $2x + 5 + x - 7$ into $3x - 2$?

The process is called “collection of like terms” or “simplifying.”
Simplify the following:

a) $x + 4 + 3x + 5$

b) $-4x + 1 + 5x - 7$

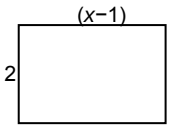
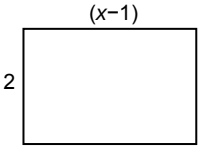
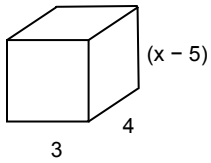
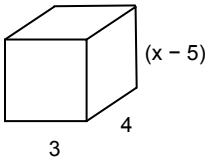
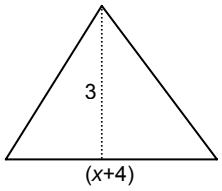
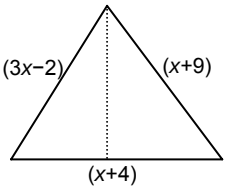
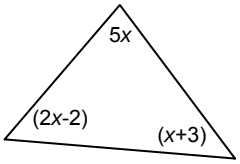
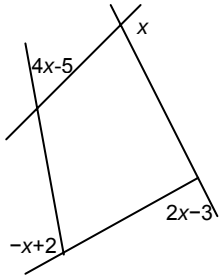
c) $5a + 2b - 2a - 6b$

d) $-2y + 4x + y + 3x$

e) $3x^2 - 5x + 6 - x^2 - 6x + 2$

5.1.5: Apply: Measurement and Geometry

Write an expression for the following. Simplify.

 Area =	 Perimeter =	 Volume =	 Surface Area =
 Area =	 Perimeter =	 Sum of the interior angles =	 Sum of the exterior angles =

5.1.6: Who Is Correct?

Reconciling Equivalent Algebraic Expressions

… How many toothpicks are needed for *n* squares?

Whose solution is correct? Show how you know.

<u>Anju's Solution</u>

"If T is the number of toothpicks and *n* is the number of squares, then, the number of toothpicks is equal to 1 plus three times the number of squares."

My equation is T = 1 + 3*n*

<u>Erin's Solution</u>

"If T is the number of toothpicks and *n* is the number of squares, then the number of toothpicks is equal to 4 plus three times one less than the number of squares."

My equation is T = 4 + 3(*n* –1).

<u>Silva's Solution</u>

"If T is the number of toothpicks and *n* is the number of squares, then the number of toothpicks is equal to 2 times the number of squares plus one more than the number of squares."

My equation is T = 2*n* + (*n* + 1).

<u>Bijuan's Solution</u>

"If T is the number of toothpicks and *n* is the number of squares, then the number of toothpicks is equal to 4 times the number of squares minus one less than the number of squares."

My equation is T = 4*n* – (*n* –1).

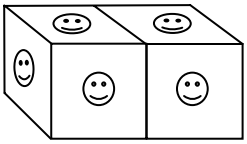
5.1.7: The Cube Sticker Problem

Problem

The Acme Toy Company makes coloured rods.
The cubes are joined in a row and a sticker machine puts “smiley” stickers on the rods.

The machine places exactly 1 sticker on the outside face of every cube. Every exposed face of each cube has to have a sticker.

This rod is 2 cubes long. It will need 10 stickers.



Procedure

Sara and Sid work together to find an algebraic model for this problem. They build a model with Cube Links and count the number of stickers needed on rods up to five cubes long.

Number of Cubes (n)	Number of Stickers (S)
1	6
2	10
3	14
4	18
5	22

Sid and Sara determined different equations to represent the relationship between the number of cubes (n) and the number of stickers (S).

Sid’s equation: $S = 4(n - 1) + 6$

Sara’s equation: $S = 10 + 4(n - 2)$

Use your knowledge of algebra to determine if Sid and Sara have the same answer. Show your work.